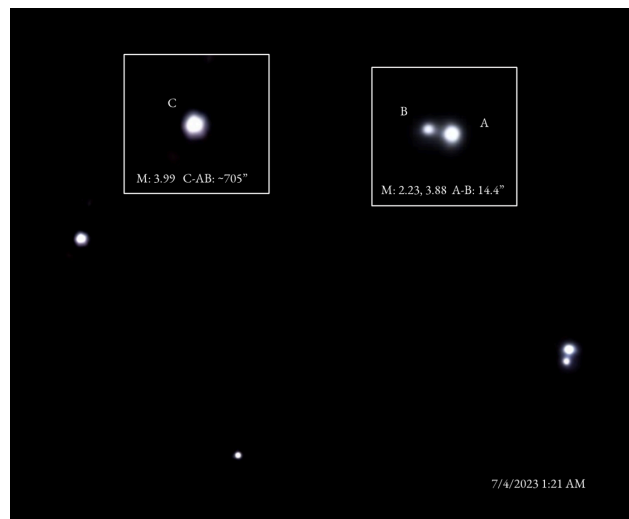
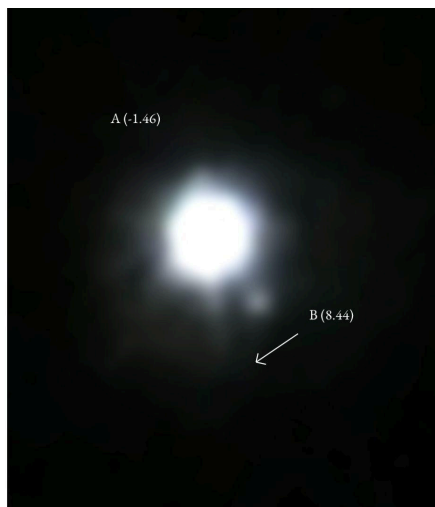
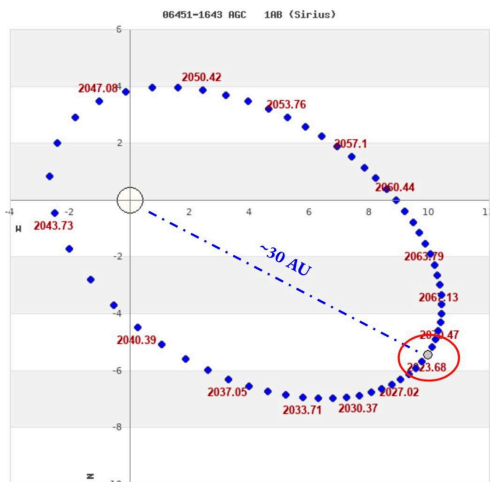


Science Olympiad CSE 2023

Novi High School

Astronomy C Test (Key)



Directions:

- **Do not open the test until told to start!**
- **Each team will be given 50 minutes to complete the test.**
- **2 sections: Section A (General Knowledge), Section BC (Calculations).**
- **Try to use 2-3 decimal places in final answers for Section BC**
- **Good luck!**

Test Written/Cover Images Credit by: Ethan Chen
(Novi '25)

Questions? Email ethankchen6@gmail.com



Name(s): **Key**

Score: **910/910**

Section A (Theory - 10 Points Each)

1. A stellar classification of V denotes a
 - a. Giants
 - b. Main Sequence**
 - c. Supergiants
 - d. Cepheids
2. Kepler's 2nd Law is derived from which law?
 - a. Law of Conservation of Momentum**
 - b. Kepler's 1st Law
 - c. Newton's Law of Gravitation
 - d. Newton's Third Law
3. What Main Sequence star has the strongest Balmer Lines?
 - a. Type O
 - b. Type A**
 - c. Type G
 - d. Type M
4. If a planet had its orbital period doubled, by what factor would its orbital radius increase?
 - a. $\sqrt{2}$
 - b. $\sqrt[3]{4}$**
 - c. 2
 - d. $\sqrt{3}$
5. All are ways to measure distances in Astronomy except:
 - a. Type Ia Supernovae
 - b. Classical/Recurrent Novae
 - c. RR Lyrae Variables
 - d. Mira Variables**

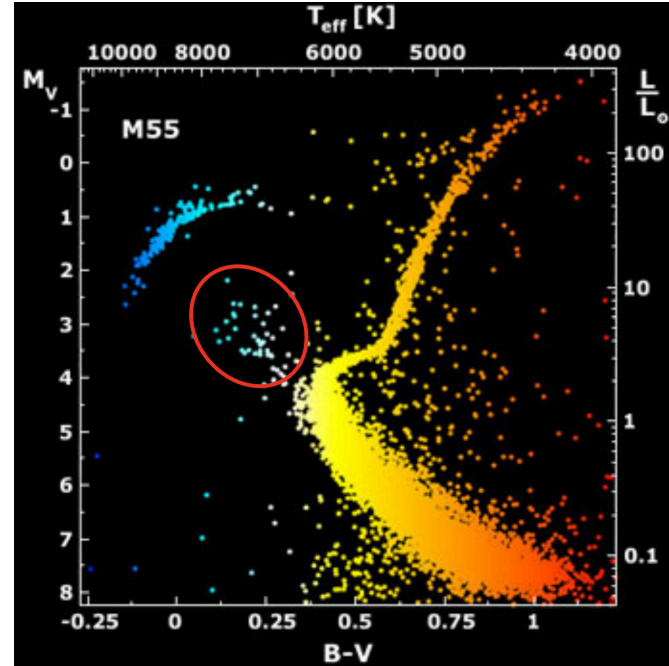
6. The Dynamical Timescale is a simplified mechanism to explain...
- a. The amount of time it takes for open star clusters to dissipate
 - b. The amount of time stars can radiate away any nuclear energy
 - c. **The amount of time stars would contract under gravity without any outward force/pressure**
 - d. The amount of time it takes for a White Dwarf to exceed the Chandrasekhar Limit
7. The Thermal Timescale is a simplified mechanism to explain...
- a. The amount of time it takes for an open star cluster to lose half of its mass
 - b. The amount of time a star can radiate energy at its Eddington Luminosity
 - c. **The amount of time it takes for a star to radiate away all kinetic energy**
 - d. The amount of time it takes for a molecular cloud to collapse into a protostar
8. All of which are methods to detect exoplanets except...
- a. Transit Method
 - b. Direct Imaging
 - c. **Doppler Curving**
 - d. Pulsar/Variable Star Timing
9. What can the radial velocity method for detecting exoplanets not tell us?
- a. Mass of the Planet
 - b. Ratio of Orbit Radii between Star/Planet
 - c. Orbital Momentum of the Planet
 - d. **Radius of the Planet**
10. In close binary star systems, mass may be transferred from one star to another or be lost completely from the system. What could be the resulting effect? Select all that apply.
- a. **Type Ia Supernova**
 - b. **Recurrent Nova**
 - c. Type II-P Supernova
 - d. **Orbital Period Decreasing**

11. The blue loop in stellar evolution occurs after what stage of evolution?
- a. Main Sequence
 - b. Planetary Nebula Phase
 - c. Subgiant Branch
 - d. Horizontal Branch**
12. “Hot Jupiter” Exoplanets are thought to be formed by:
- a. Collision between two rocky or gas planets inside the frost line
 - b. Jupiter sized planet losing its atmosphere from stellar wind
 - c. Rocky planet accreting an atmosphere of mainly carbon/hydrogen based compounds
 - d. Jupiter sized planets “migrating” towards the parent star**
13. For an object that is gravitationally bound to another, the orbital energy is always:
(select all that apply):
- a. Positive
 - b. Negative**
 - c. Constant**
 - d. Varies at points in the orbit
14. Type Ib and Ic Supernova lack the emission line of what element?
- a. Carbon
 - b. Neon
 - c. Hydrogen**
 - d. Silicon
15. How can Type Ib or Ic Supernovae occur instead of a Type II Supernova?
- a. Binary Star Interaction with close companion**
 - b. Ignition of Helium Fusion in Core
 - c. WR Stars expelling outer layers through stellar wind**
 - d. Ignition of Iron/Magnesium Core Fusion

16. The detection of metals (such as lithium) in Population I stars can be attributed to
- Classical/Recurrent Novae**
 - Type Ia/II Supernovae
 - Asymptotic Giant Branch Stars
 - Incorrect models of globular clusters

Questions 17-20 relate to the Hertzsprung-Russell Diagram of the globular cluster M55.

17. What B-V color index and absolute magnitude respectively would RR Lyrae variables be found?
- 0.20/3.50
 - There are no RR Lyrae Variables in this cluster
 - 0.3/0.00**
 - 1.00/-1.00



18. Why is the Main-Sequence Turnoff Point Important?
- Enables astronomers to determine metallicity of the cluster
 - Enables astronomers to determine the age of the cluster**
 - Astronomers can use the turnoff point to determine the amount of stars in the cluster
 - Main-Sequence Turnoff point can show astronomers the half-light radius of the cluster
19. At approximately what solar luminosity does the Main Sequence Turn-Off Point occur (Probably inaccurate due to interstellar extinction)?
- $0.5 L_{\odot}$
 - $5 L_{\odot}$
 - $2 L_{\odot}$ (This is the answer, I cant bold the solar luminosity symbol)**
 - $8 L_{\odot}$

20. The stars circled in red on the diagram are called blue stragglers. How are these stars formed?
- a. Stars that formed with a higher mass than the rest of the cluster
 - b. Merger of two stars in a binary system**
 - c. Stars that have lost mass in stellar wind to continue on the main sequence longer
 - d. White Dwarf accreting material from companion star
-
21. All Transiting Exoplanets and Eclipsing Binary Star Systems have inclination angles of approximately
- a. 0°
 - b. 90° (Answer)**
 - c. 45°
 - d. 180°
22. During a helium flash in low-intermediate mass stars, why does the overall luminosity decrease?
- a. Helium Flash causes radius to decrease and temperature to increase (Due to star seeking new equilibrium after helium flash)**
 - b. The fusion of hydrogen in a shell is disrupted
 - c. Helium Flash does not lower the overall luminosity
 - d. The overall luminosity decreases because the onset of helium fusion transfers energy away from the outer layers of the star
23. The “plateau” seen in a Type II-P Supernova is caused by
- a. Recombination of Helium ejected from stellar wind
 - b. Opaque Ionized Hydrogen**
 - c. Fusion of more heavier elements during the supernova
 - d. Opaque Ionized Silicon decaying into Nickel
24. Why do low mass stars have radiative cores while high mass stars have convective cores?
- a. Low mass stars do not have a core hot enough to be convective
 - b. Low mass stars have a low temperature gradient**
 - c. High mass stars have a core too hot to be radiative
 - d. High mass stars have a low temperature gradient

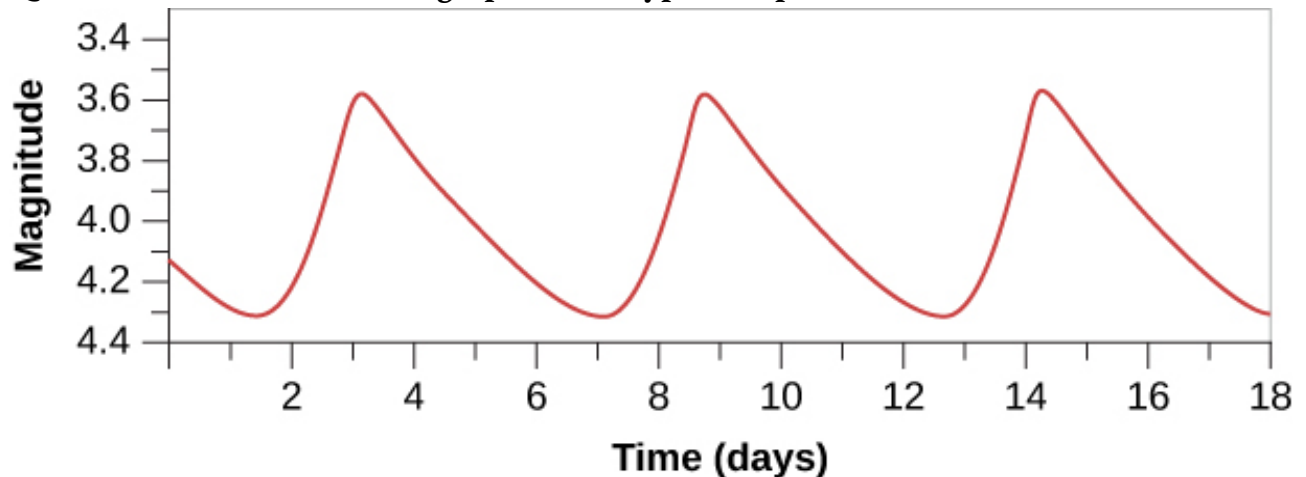
25. Stars move onto the Asymptotic Giant Branch track of the HR diagram when:

- a. Hydrogen begins to run out in the core
- b. The onset of helium fusion in the core
- c. Fusion of hydrogen in a shell around the core stops
- d. Helium begins to run out in the core**

26. Why can't stars fuse iron?

- a. The high binding energy of iron makes iron fusion endothermic**
- b. Iron is unstable inside the cores of stars
- c. Fusion of heavier elements in other shells prevents stable fusion of iron
- d. Iron decays immediately into Nickel

Questions 27-31 relate to the graph of the Type II Cepheid Variable Star shown below



27. By how many magnitudes are Type II Cepheids higher than Type I Cepheids?

- a. 3.0
- b. 1.5**
- c. 1.0
- d. 0.5

28. If the light curve shown was of a Type I Cepheid, what would its average absolute magnitude be?

- a. -2.5
- b. -3.0
- c. -3.5**

d. -4.0

29. Why are Type II Cepheids less luminous than Type I Cepheids?

- a. Type I Cepheids have more helium in their photosphere
- b. Type II Cepheids use a different pulsation method
- c. Type I Cepheids are older
- d. **Type II Cepheids are older**

30. Cepheid variables are thought to be going through the

- a. Horizontal Branch
- b. Asymptotic Giant Branch
- c. Giant Branch
- d. **Blue Loop**

Section B: Short Answer (15 Points Each)

(135 Points Total)

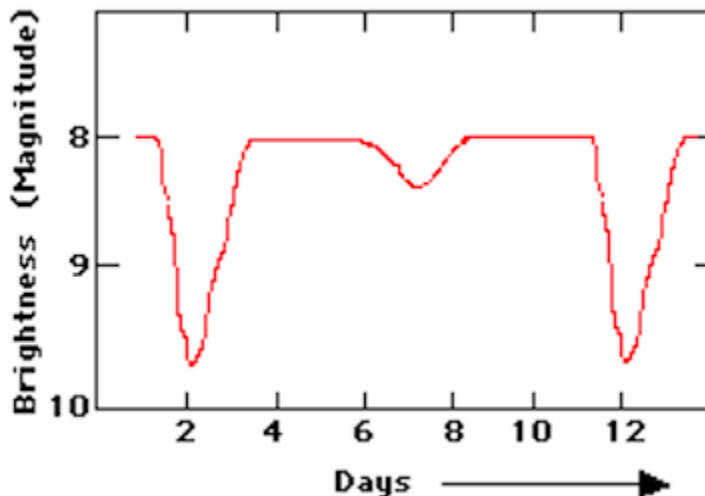
31. What is period of the Cepheid light curve shown?

Accept 5.3 - 5.8 Days

32. How are Type Ia Supernovae different from Classical/Recurrent Novae

Type Ia Supernovae: Carbon Detonation, White Dwarf is destroyed

Classical/Recurrent Novae: Runaway Hydrogen/Helium on surface, White dwarf is not destroyed



Questions 33-35 relate to the graph shown

33. What object is the graph depicting?

Eclipsing Binary Stars

34. By what factor does the luminosity change from no eclipse to the primary eclipse?

Accept 5x- 5.5x

35. By what factor does the luminosity change from no eclipse to the secondary eclipse?

Accept 1.2x - 1.5x

36. What causes stars to undergo a blue loop on the HR diagram in stellar evolution?

After the reaching the tip of the red giant branch and igniting helium fusion in the core, the star moves onto the horizontal branch, moving slightly down and to leftwards on the HR Diagram – increasing temperature

37. At what points (Lowest/Highest Luminosity) of a Cepheid Pulsating Variable is helium ionized?

Lowest Brightness: Ionized Helium

Lowest Brightness: Un-Ionized Helium

38. Explain why the radial velocity method for detecting exoplanets will not always give an accurate value for the exoplanet's mass

Orbital inclination can affect radial velocity readings by making them lower than measured. A 90 degree inclination is ideal for an edge-on view of radial velocity

39. What is the Eddington Luminosity and why is it important?

Maximum luminosity a star can obtain without blowing itself apart - Can explain the high stellar mass loss of massive stars/limits to accretion disks

40. True or False, during outbursts the bolometric luminosity of Luminous Blue Variables remains constant. **True**

Section BC (Calculations)

1. Kepler's Laws and Virial Theorem are two very important laws in astrophysics that define countless other laws and theorems, but how are they derived?
 - a. Write a derivation of Kepler's Third Law relating orbital period T and orbital radius A (Hint: Centripetal Force with angular velocity and Gravitational Force). Note: this is only the derivation for a **circular** orbit **(35 Points)**

1. Relate the net force on an orbiting particle, setting centripetal force equal to gravitational force. **(10 Points)**

$$m\omega^2 r = G \frac{Mm}{r^2}$$

2. Rewriting angular velocity ω as $\frac{2\pi}{T}$. **(10 Points)**

$$m\left(\frac{2\pi}{T}\right)^2 r = G \frac{Mm}{r^2}$$

3. Eliminating the orbiting point mass m , and expanding $\left(\frac{2\pi}{T}\right)^2$ **(5 Points)**

$$\frac{4\pi^2}{T^2} r = G \frac{M}{r^2}$$

4. Rearranging terms yields. **(10 Points – must be written with terms T and a , and must show at least 2 of the above steps. 5 Points if showing only 1 of the above steps)**

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

- b. Virial Theorem states that for any gravitationally bound system, the average kinetic energy is equal to the $-\frac{1}{2}$ the average gravitational potential energy. Show how centripetal force/gravitation are related to kinetic/gravitational energy **(15 Points)**

The equation for Gravitational PE is provided below

$$U = -\frac{GMm}{R}$$

1. Relating Centripetal Force to Gravitational Force in a system. **(5 Points)**

$$\frac{mv^2}{r} = G \frac{Mm}{r^2}$$

2. Multiplying by r to both sides and showing how kinetic energy ($\frac{1}{2}mv^2$) is related by multiplying both sides by $\frac{1}{2}$. **(10 Points)**

$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{GMm}{r}$$

- c. Rewrite your answer above in terms of Kinetic Energy **T** and Gravitational Potential Energy **U** **(15 Points)**

1. Rewriting in terms of **T** and **U** with the form $T = -\frac{1}{2}U$, with the appropriate negative sign. **(15 Points)**

$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{GMm}{r} = \frac{1}{2} \left(- \left(- \frac{GMm}{r} \right) \right)$$

$$T = \frac{1}{2} (-U) \rightarrow T = -\frac{1}{2}U$$

- d. From the first step in the derivation of Kepler's Third Law, derive the equation for pure circular orbital velocity in terms of v (Hint: What is another form to write centripetal force in?)
(20 Points)

1. Writing Centripetal Force as $\frac{mv^2}{r}$ instead of $m\omega^2 r$. **(10 Points)**

$$\frac{mv^2}{r} = G \frac{Mm}{r^2}$$

2. Eliminate orbiting point mass m and multiplying out r . **(5 Points)**

$$v^2 = \frac{GM}{r}$$

3. Writing in terms of v (must include square root) **(5 Points)**

$$v = \sqrt{\frac{GM}{r}}$$

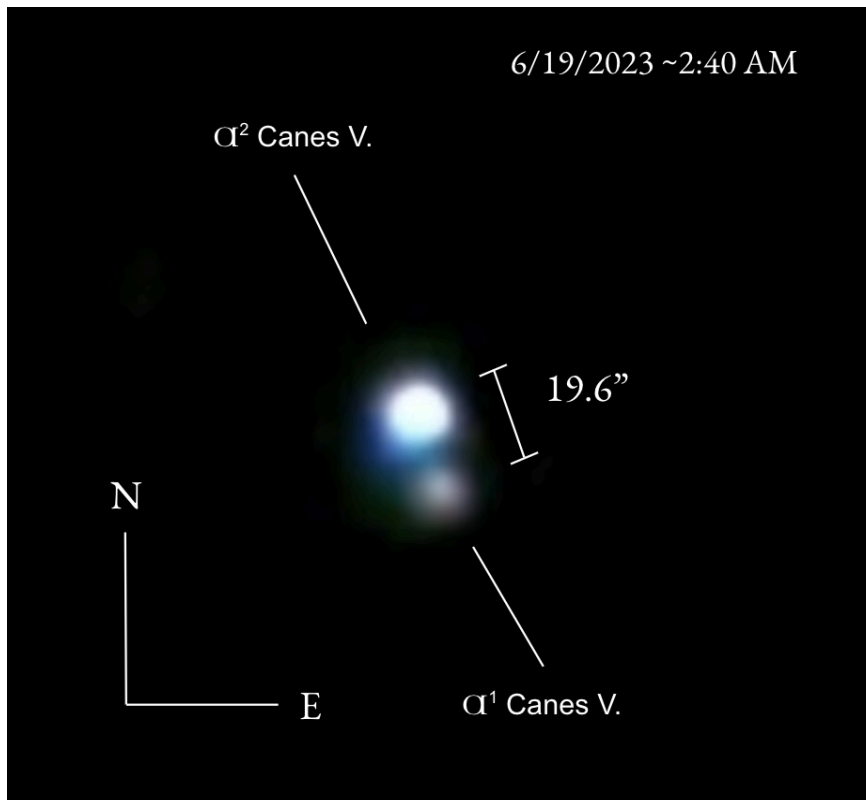
2. Double stars have fascinated early astronomers as they appeared as a single point of light without a telescope and they enabled astronomers to measure proper motions relative to another star. Some double stars are gravitationally related while others are optical “line of sight” binaries.

- [Cor Caroli](#) (α Canum Venaticorum) is a double star (image taken by test author shown) in the constellation of Canes Venatici. Information about the two stars is given in the table below

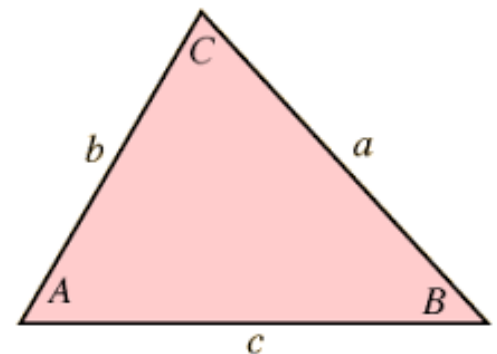
Star	Mass	Spec. Type	Proper Motion (Dec)	Proper Motion (RA)	Radial Velocity	Distance
α^2 C.V.	$2.97 M_{\odot}$	A0V	$53.54 \frac{mas}{yr}$	$-235.08 \frac{mas}{yr}$	$-4.10 \frac{km}{s}$	100 ly
α^1 C.V.	$1.47 M_{\odot}$	F2V	$55.69 \frac{mas}{yr}$	$-232.86 \frac{mas}{yr}$	$-0.60 \frac{km}{s}$	106.5 ly

Proper Motion Equation: $v_t = 4.7406 \mu d$

$[\mu] = "/yr$
 $[d] = \text{parsecs}$
 $[v_t] = \text{km/s}$



$$c^2 = a^2 + b^2 - 2ab \cos C$$



- a. Hydrogen Alpha Lines (electrons falling from 3rd to 2nd energy level in H) have a wavelength of approximately 656.28 nm. What is the change in wavelength ($\Delta\lambda$) due to blue/redshift for both stars? Is it a red or blue shift? **(25 Points)**

Using the equation for blue/redshift

$$\frac{\Delta\lambda}{\lambda_{\text{Emitted}}} = \frac{v}{c}$$

α^2 C.V. **(10 Points)**

$$\frac{\Delta\lambda}{656.28 \text{ nm}} = \frac{-4.10 \text{ km/s}}{3 \times 10^5 \text{ km/s}}, \Delta\lambda = -0.00896916 \text{ nm}$$

α^1 C.V. **(10 Points)**

$$\frac{\Delta\lambda}{656.28 \text{ nm}} = \frac{-0.60 \text{ km/s}}{3 \times 10^5 \text{ km/s}}, \Delta\lambda = -0.00131256 \text{ nm}$$

Stating that it is blueshift for both stars (5 Points)

- b. The stellar components of Cor Caroli share a common proper motion (different from an orbit), but do they form a gravitationally bound orbit? Using the data and information given, determine if Cor Caroli is gravitationally bound in an orbit. **(70 Points)**

Calculate the distance between the two stars using the law of cosines (10 Points)

$$d^2 = (100)^2 + (106.5)^2 - 2(100)(106.5)\cos\left(\frac{19.6}{3600}\right), d = 6.50 \text{ ly}$$

Then, determine both star's proper motion in "/year **(10 Points EACH)**

α^2 C.V.

$$\sqrt{(0.05354)^2 + (-0.23508)^2} = 0.2411'' \frac{''}{yr}$$

α^1 C.V.

$$\sqrt{(0.05569)^2 + (-0.23286)^2} = 0.2394'' \frac{''}{yr}$$

To find the star's true velocity through space, we need to combine the proper motion of the star with its radial velocity ([explanation](#))

(10 Points Each)

α^2 C.V.

$$\sqrt{\left(-4.10 \frac{km}{s}\right)^2 + \left[4.7406(0.2411 \frac{''}{yr})\left(\frac{100}{3.26} ly\right)\right]^2} = 35.299 km/s$$

α^1 C.V.

$$\sqrt{\left(-0.60 \frac{km}{s}\right)^2 + \left[4.7406(0.2394 \frac{''}{yr})\left(\frac{106.5}{3.26} ly\right)\right]^2} = 37.081 km/s$$

Finding their relative velocity to be **(5 Points)**

$$v_{rel} = 37.081 km/s - 35.299 km/s = 1.782 km/s$$

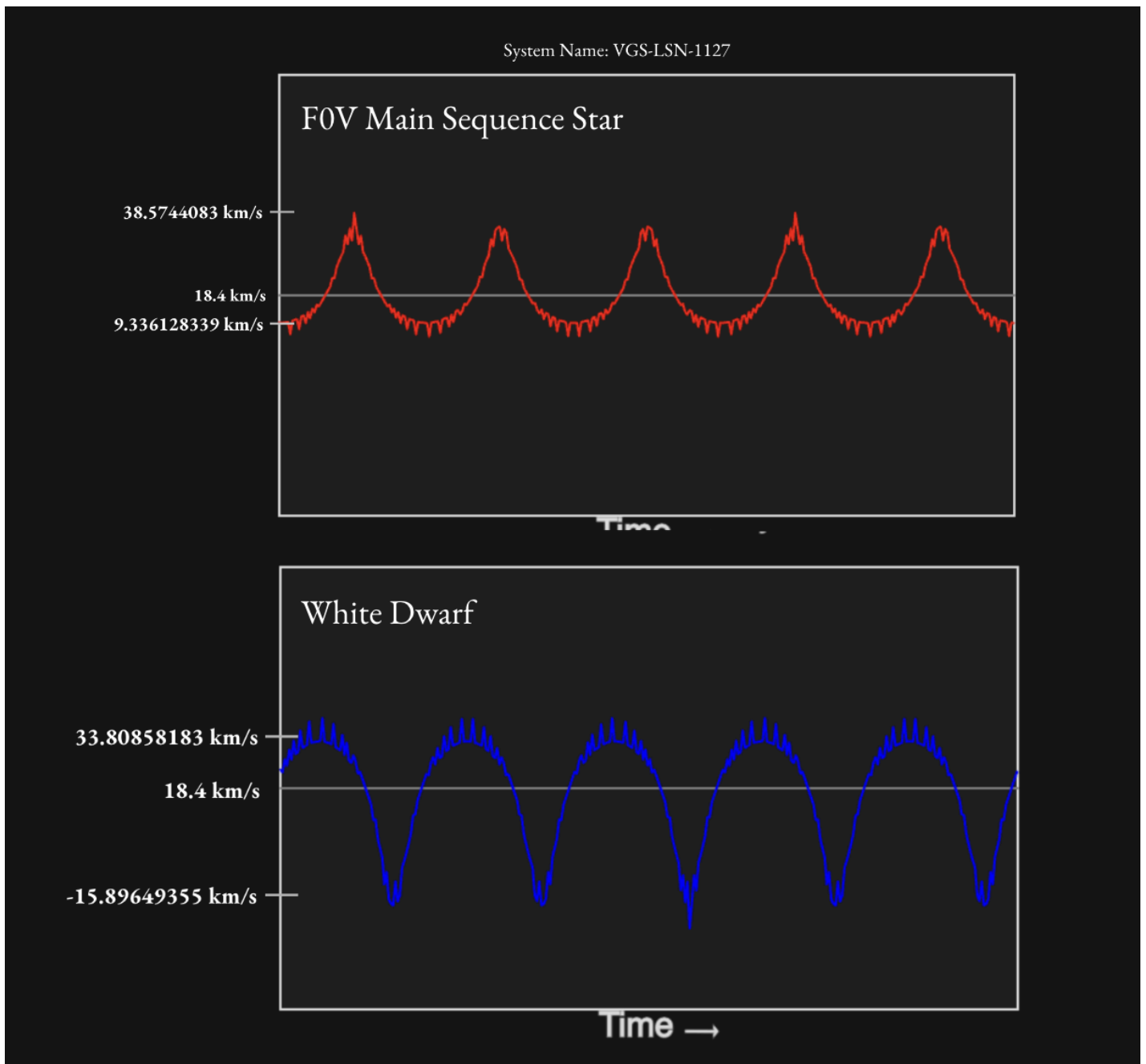
To determine if they are gravitationally bound in an orbit, find their relative escape velocity **(10 Points)**

$$v_{esc, relative} = \sqrt{\frac{2G(2.97 + 1.47)(1.989 \times 10^{30} kg)}{\frac{6.50}{9.46 \times 10^{15}}}} \rightarrow 0.13846 km/s$$

Because $v_{rel} > v_{esc}$, Cor Caroli is not gravitationally bound in an orbit and exists as a common proper motion double star **(5 Points)**

3a. A F-Class Main Sequence Star with a mass of $1.7 M_{\odot}$ and a White Dwarf Star have their radial velocity curve shown below. Using radial-velocity curve and data given below, answer the following questions

- Orbital Inclination: 66° (**Important for radial velocity**)
- Orbital Period: 1.18678792 yr
- A distance of 0.93 AU between the two stars at perihelion



- a. Is this star system moving away or towards the observer? How can you tell?
(10 Points)

The star system is moving away from the observer. (5 Points) Radial Velocity of the center of mass is 18.4 km/s - positive radial velocity means moving away from us (redshift) (5 Points)

- b. On the orbital diagram below, label the following (5 Points Each)
- Closest approach of 0.93 AU
 - Arrows denoting the direction of their velocities and movement of the barycenter
 - Value of the velocity that the barycenter is receding from the observer
 - Not required to find the velocities of each star

Figure 1

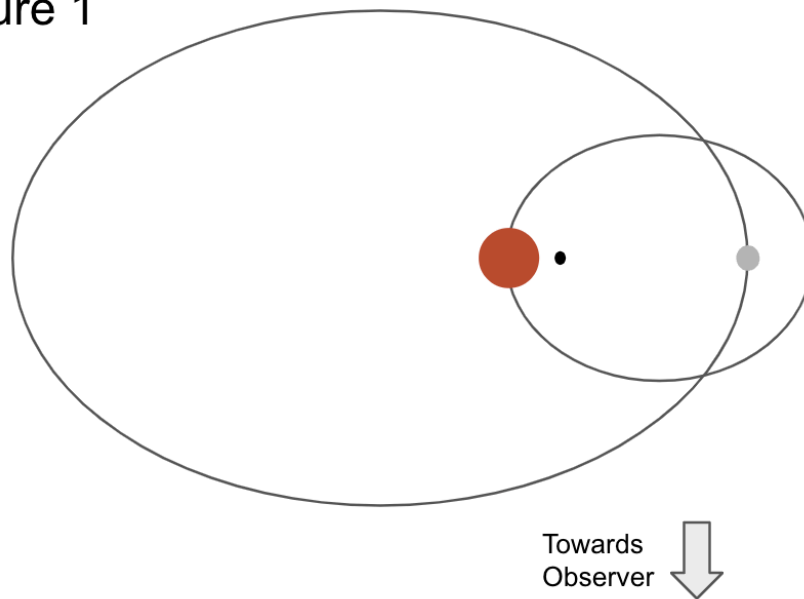
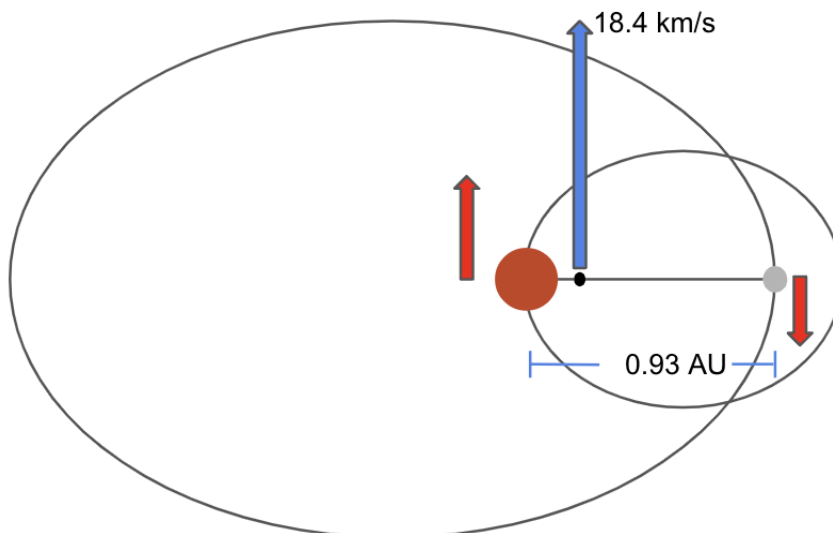


Figure 1 (Key)



(5 Points) Each for labeling correct direction of motion of BOTH stars, direction of motion of center of mass and separation between stars. 15 points total.

- c. What is the velocity (in km/s) of the F-Main Sequence Star and White Dwarf in km/s at Aphelion and Perihelion **relative to the barycenter**? (Include correct negative signs) **(40 Points)**

Key Point: Because the orbital inclination is not 90° , radial velocity will be less than observed. $\frac{v_{observed}}{\sin(i)} = v_{actual}$. And because the barycenter is receding from the observer at 18.4 km/s, we must subtract that value from any radial velocity we measure.

F-Main Seq:

- **At Perihelion:** $\frac{38.5744083 - 18.4 \text{ km/s}}{\sin(66^\circ)} = 22.08363922 \text{ km/s}$
(10 Points)
- **At Aphelion:** $\frac{18.4 - 9.336128339 \text{ km/s}}{\sin(66^\circ)} = -9.921642744 \text{ km/s}$
(10 Points)

White Dwarf:

- **At Perihelion:** $\frac{-15.89649355 - 18.4 \text{ km/s}}{\sin(66^\circ)} = -37.54218606 \text{ km/s}$
(10 Points)
- **At Aphelion:** $\frac{33.80858183 - 18.4 \text{ km/s}}{\sin(66^\circ)} = 16.86679267 \text{ km/s}$
(10 Points)

d. What is the mass (in solar masses) of the White Dwarf Companion?

(15 Points)

$$m_1 v_1 = m_2 v_2, m_1 = 1.7 M_{\odot}, \text{Key uses Aphelion Velocities}$$

$$m_2 = (1.7 M_{\odot}) \left(\frac{|-9.921642744 \text{ km/s}|}{16.86679267 \text{ km/s}} \right) = 1.0 M_{\odot} \text{ (15 Points)}$$

e. What is the combined semi major axis (in AU) of the system?

(15 Points)

$$P^2 = \frac{a^3}{M}$$

$$(1.18678792 \text{ yr})^2 = \frac{a^3}{1.7 + 1.0 M_{\odot}}, a = 1.50 \text{ AU} \text{ (15 Points)}$$

f. What is the farthest separation (in AU) between the two stars?

(45 Points)

Solve individual Semi Major Axes

$$m_1 a_1 = m_2 a_2 \rightarrow a_2 = \frac{m_1}{m_{total}} (a_1 + a_2)$$

$$a_1 = 0.5556 \text{ AU}, a_2 = 0.9444 \text{ AU} \text{ (15 Points for both SMA's)}$$

Solve for eccentricity knowing the closest approach of 0.93 Au

$$0.5556(1 - e) = 0.93 \text{ AU} \left(\frac{1.0 M_{\odot}}{2.7 M_{\odot}} \right), e = 0.38$$

(10 Points for correct eccentricity)

$$r_{farthest} = 0.5556(1 + 0.38) + 0.9444(1 + 0.38) = 2.07 \text{ AU}$$

(15 Points)

3b. The F0V Main Sequence Star evolves off the Main Sequence onto the Red Giant Branch. Due to the gravitational pull of the White Dwarf Companion, the primary star loses mass at a constant rate of $10^{-6} \frac{M_{\odot}}{\text{yr}}$. 20% of the stellar mass loss is accreted (constantly) onto the White Dwarf. **(145 Points Total)**

- a. How long (in millions of years) will it take for the white dwarf to exceed the Chandrasekhar Limit? **(10 Points)**

$$\text{amount} = \text{rate}(\text{time}) + \text{start amount}$$

$$1.44 M_{\odot} = (0.20(10^{-6} \frac{M_{\odot}}{\text{yr}}))t + 1.00 M_{\odot}, t = 2.2 \text{ million years}$$

- b. What is the rate of loss of energy (in J/s) the star system is losing from the stellar mass loss? Assume the change in the orbital distances caused by the mass loss to be negligible (for now). **(30 Points)**

Total Energy of the System:

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - G \frac{m_1 m_2}{R} = - \frac{G m_1 m_2}{2a}$$

Take derivative of energy of system (with respect to time), knowing that change in any orbital distances is negligible

$$\frac{dE}{dt} = \frac{d}{dt} \left[- G \frac{m_1 m_2}{2a} \right] = - \frac{G}{2a} \frac{d}{dt} [m_1 m_2] = - \frac{G}{2a} \left(m_1 \frac{dm_2}{dt} + m_2 \frac{dm_1}{dt} \right)$$

Primary Star loses $10^{-6} \frac{M_{\odot}}{\text{yr}}$, 20% is accreted onto the White Dwarf

$$\frac{dE}{dt} = L = - \frac{G}{2[1.5 \text{ AU}]} ([1.7 M_{\odot}][0.20 \cdot 10^{-6} \frac{M_{\odot}}{\text{yr}}] + [1.0 M_{\odot}][- 10^{-6} \frac{M_{\odot}}{\text{yr}}])$$

(Convert to SI units)

$$= 1.231 \cdot 10^{25} \frac{\text{J}}{\text{s}} \rightarrow 0.0322 L_{\odot} \text{ **(30 Points for answer in J/s)}**$$

- c. From an accreting mass m , formulate an expression for the energy generated for the accreting mass falling a distance R . Assume the white dwarf's radius r is negligible. **(15 Points)**

For the energy released by a mass m falling a distance R towards the White Dwarf m_2 with a radius r

$$E = - \left(G \frac{m_2 m}{r} - G \frac{m_2 m}{R} \right), \text{ radius of white dwarf is negligible}$$

$$E = G \frac{m_2 m}{R}$$

- d. What is the rate of energy generation produced by the accreting matter in solar luminosities? Assume an average accretion distance of 1.5 AU (the combined semi major axes). Assume the mass of the White Dwarf m_2 does not change significantly with a small mass accreted m . Answer in L_{\odot}

(30 Points)

Once again taking the time derivative of energy gives luminosity

$$\frac{d}{dt} \left[G \frac{m_2 m}{R} \right] = G \frac{m_2}{R} \frac{dm}{dt}, \text{ we can assume } m_2 \text{ remains constant for a}$$

small mass dm that is accreted per unit time - This is so we don't have to derivative it.

$$G \frac{m_2}{R} \frac{dm}{dt} = G \frac{[1.0 M_{\odot}]}{[1.5 AU]} (0.20 \cdot [10^{-6} \frac{M_{\odot}}{yr}]) \rightarrow 0.0195 L_{\odot}$$

(30 Points for answer in solar luminosities)

- e. What is the rate that the closest approach (in m/s) between the 2 stars of 0.93 AU increases/decreases? (Hint: Use the answer from question b and orbital energy conservation) **(60 Points)**

Another derivative question

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - G\frac{m_1m_2}{r} = -G\frac{m_1m_2}{2a}$$

(20 Points for subbing in value from question b)

$$\frac{d}{dt} \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - G\frac{m_1m_2}{r} \right] = \frac{d}{dt} \left[-G\frac{m_1m_2}{2a} \right] = 1.231 \cdot 10^{25} \frac{J}{s}$$

$$\frac{1}{2}v_1^2 \frac{d}{dt} [m_1] + \frac{1}{2}v_2^2 \frac{d}{dt} [m_2] - G \frac{d}{dt} \left[\frac{m_1m_2}{r} \right] = 1.231 \cdot 10^{25} \frac{J}{s}$$

$$\frac{1}{2}v_1^2 \frac{d}{dt} [m_1] + \frac{1}{2}v_2^2 \frac{d}{dt} [m_2] - G \left(\frac{(m_1 \frac{dm_2}{dt} + m_2 \frac{dm_1}{dt})r - \frac{dr}{dt}(m_1m_2)}{r^2} \right) = 1.231 \cdot 10^{25} \frac{J}{s}$$

Using velocities at perihelion and plugging in (CONVERT TO SI UNITS):

$$v_1 = 22.0836 \text{ km/s}, v_2 = 37.542 \text{ km/s}, r = 0.93 \text{ AU}$$

$$\frac{d}{dt} [m_1] = -10^{-6} \frac{M_\odot}{\text{yr}}, \frac{d}{dt} [m_2] = (0.20(10^{-6} \frac{M_\odot}{\text{yr}}))$$

The closest approach distance **DECREASES at a rate of**

$$\frac{dr}{dt} = -9.0195 \cdot 10^{-4} \text{ m/s}$$

(40 Points for correct answer)