Science Olympiad CSE 2024
Astronomy C
Novi High School

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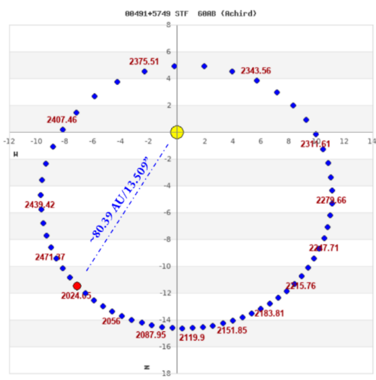
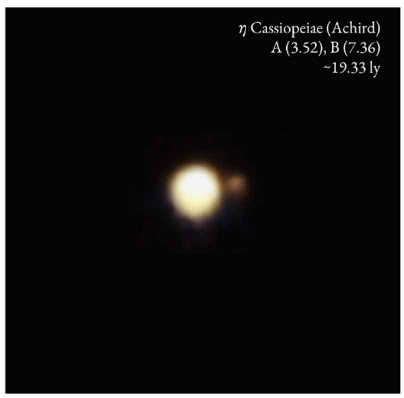
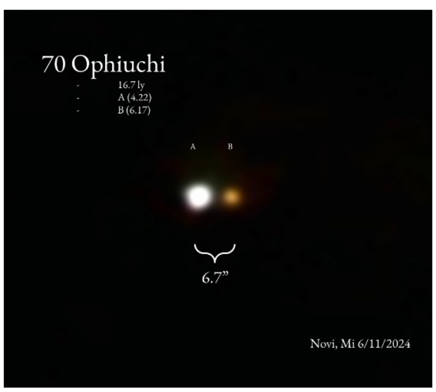
Score: /315







1 Multiple Choice

2 points per question

1. Which of these is **NOT** a measure of **astronomical** distance?
   (a) Parsecs
   (b) Lightyears
   (c) Astronomical units
   (d) Hertz

2. Nuclear fusion in the Sun works by fusing which 2 atoms of which element?
   (a) Hydrogen
   (b) Helium
   (c) Carbon
   (d) Calcium

3. Which of these can be used to determine the atmospheric composition of an exoplanet?
   (a) Transit spectroscopy
   (b) JS9
   (c) Albedo
   (d) None of the above

4. Which of these is **NOT** normally determined directly from an H-R Diagram ?
   (a) Age
   (b) Luminosity
   (c) Temperature
   (d) Trace elements

5. What are the limiting constraints of the Jeans Mass?
   (a) It assumes a constant density
   (b) It assumes the cloud is half hydrogen and half dust
   (c) It assumes that it follows the laws of thermodynamics
   (d) It assumes that all matter is ionized in hot star formation

6. An excess of infrared radiation around a star usually indicates:
   (a) Low Mass Companion/Brown Dwarf
   (b) Cloud of cold molecular hydrogen
   (c) Dust/Debris Disk
   (d) Accretion of Gas

7. Hydrostatic Equilibrium can be achieved through the following means except:
   (a) Electron Degeneracy Pressure
   (b) Radiation Pressure
   (c) Thermal Pressure
   (d) Convecting Pressure

8. What exoplanet detection method can work in tandem with the transit method?
   (a) Speckle Photometry
   (b) Timing Variations
   (c) Astrometry
   (d) Microlensing

9. A key difference between albedo and emissivity is
   (a) Albedo deals with mostly with reflection in the optical spectrum of light while emissivity deals with thermal radiation.
   (b) Albedo is a measure of reflection while emissivity is a measure of absorbance
   (c) Albedo measures reflection of thermal radiation while emissivity measures absorbance of optical wavelengths.
   (d) There is no difference.

10. Launched in 2009, the Kepler Space Telescope found exoplanets in a “field” of stars around what constellations?
    (a) Cygnus, Sagitta, Draco
    (b) Cygnus, Delphinus, Aquila
    (c) Cygnus, Lyra, Cassopeia
    (d) Cygnus, Lyra, Draco

2 Short Answer

2.1 Stellar Evolution

2.1.1 Pre-Main Sequence

1. What track can PMS stars follow if they have mass less than 3 solar masses? [2 point]

   Hayashi Track

2. A certain type of PMS star contracting around the Hayashi track is commonly found near molecular clouds. State the type of star it is, then tell which metal is burned by the proton-proton chain. [3 points]

   T Tauri Star (2 point), Lithium (1 point)

3. Name the other path PMS stars can take if they have a mass above 0.5 solar masses. How does luminosity change as a star following this sequence approaches the main sequence? [3 points]

   Henyey Track (2 point), luminosity remains close to constant (1 point)

2.1.2 Apple Grape Banana (AGB)

1. What does the acronym AGB stand for in stellar evolution? [2 point]

   Asymptotic Giant Branch

2. How does energy generation during the AGB phase compare to that of red giant evolution? [2 point]

   Faster energy generation (1 point), but lasts only a short amount of time (1 point)

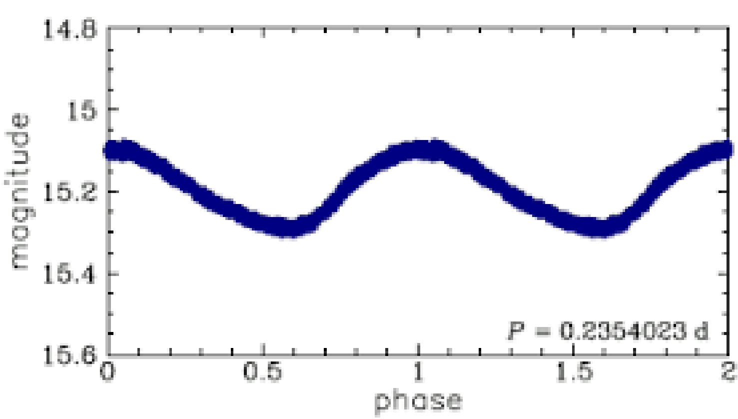
3. What becomes of the central star post-AGB? [2 point]

   Cools to a white dwarf

2.2 Variables

2.2.1 RR Lyrae

1. An RR Lyrae light curve is shown below. Which type of RR Lyrae Variable is it? [2 points]



   RRc variable

2. For the above type of RR Lyrae variable, write approximately what percent of RR Lyrae variables fall under this classification. Is the period shorter or longer than other types of RR Lyrae variables? [4 points]

   9 percent (2 points), shorter periods (2 points)

3. Describe the light curve of the other type(s) of RR Lyrae variables. Are they more or less common than the RR Lyrae variable type shown above? [5 points]

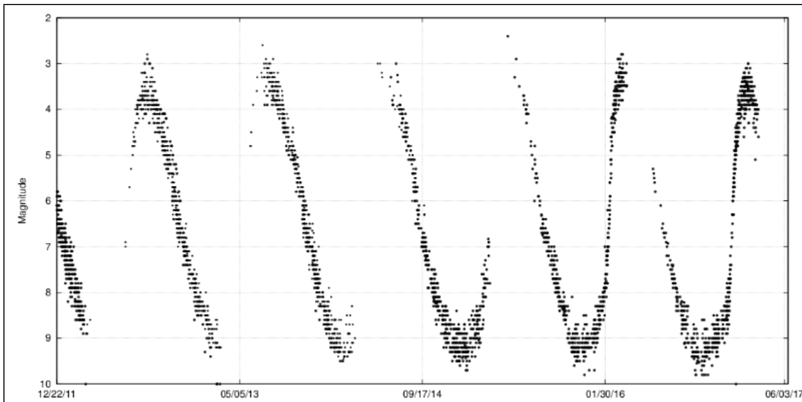
   RRa and RRb have steep rises in brightness, rather than a sinusoidal light curve (4 points), much more common (1 point)

4. RR Lyrae variables are classified as standard candles. Explain what a standard candle is, and tell the name of another type of star that can be used as a standard candle. [5 points]

   Objects that make up the "cosmic distance ladder" and are used to measure distances to farther galaxies (3 point), accept any Cepheid variable, Delta Scuti (2 points maximum)

2.2.2 Mira Variables

1. In the box below, draw a light curve that would be typical of a Mira variable. Be sure to include an appropriate pulsation period. [3 points]



Lithopsian, CC BY-SA 4.0, Wikimedia Commons

2. What are dredge-ups, and what causes them? [4 points]

   Periodic deep convection (2 point), fusion in alternating hydrogen and helium shells (2 point)

3. In solar masses, what is the approximate maximum mass of a Mira variable? [1 point]

   2 $M_{\odot}$

4. What causes some Mira variables to appear to change their period over time? [3 points]

   Thermal pulses (2 points), helium reignites outer hydrogen shell (1 point)

2.3 Hertzsprung-Russell Diagrams

1. Fill out the table below based on the types of stars found in regions of an HR diagram: [5 points]

I	Supergiants (1 points)
II	Bright Giants (1 points)
III	Giants (1 points)
IV	Subgiants (1 points)
V	Main sequence (1 points)

2. If a star were following the Hayashi track, how would it be traveling on an HR diagram? [3 points]

   Nearly vertical (2 point), close to constant temperature (1 point)

3. If a star were following the Henyey track, how would it be traveling on an HR diagram? [3 points]

   Nearly horizontal (2 point), close to constant luminosity (1 point)

4. What does a white dwarf star's position on the HR diagram say about its temperature and luminosity? [3 points]

   High temperatures (1 point), but low luminosity due to small size (2 point)

3 Astrophysics

3.1 Low Mass Stellar Companion (80 Points)

Large gas exoplanets may be erroneously categorized as brown dwarfs as it is difficult to distinguish between them in forming star systems. A supposed exoplanet in a binary system named **Novi C12** is observed in a circular orbit 16.3 light years away. The parent star has a mass of 0.579 $M_\odot$ with a radial velocity amplitude of 1300 m/s.

1. The orbital period of the system is recorded as 2.315 years and the orbit is inclined at 60 degrees. What is the mass of the secondary object (Novi C12 B) in M_J? [10 Points]

$$\frac{{m_2}^3 sin^3(i)}{(m_1+m_2)^2} = \frac{P{K_1}^3}{2\pi G}$$

$$\frac{{m_2}^3 sin^3(60)}{(0.579[M_\odot]+m_2)^2} = \frac{(2.315[years])\cdot(1300m/s)^3}{2\pi G} \rightarrow m_2 = 51.224M_J$$

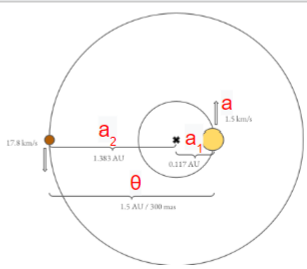
2. At maximum angular separation, the two bodies are observed to be $\theta = 300\ mas$, what is the the physical separation between each star and the barycenter in AU? [10 Points]

300 milliarcseconds = 0.3 arcseconds
$\theta[rads] = \frac{diameter}{distance} \rightarrow \theta[arcsecs] = 206265\frac{diameter}{distance}$

$$0.3arcseconds = 206265\frac{a}{16.3[ly]\cdot 9.46\cdot 10^{15}[m]}, a = 2.24\cdot 10^{11}m$$

a=1.5 AU
$a_1 = 1.5AU(\frac{51.224M_J}{51.224M_J+0.579M_\odot}) = 0.117AU$
$a_2 = 1.5AU(\frac{0.579M_\odot}{51.224M_J+0.579M_\odot}) = 1.383AU$

3. Because this system involves only two bodies, we can solve the parameters of the system. Label θ, a, a_1, a_2, and the true velocities v_1 and v_2 adjusted for orbital inclination. [15 Points]



$$v_1 = \frac{2\pi(0.117AU)}{2.315[years]} = 1.5km/s$$

$$v_2 = \frac{2\pi(1.383AU)}{2.315[years]} = 17.8km/s$$

4. Brown Dwarfs, for a brief period, gravitationally contract over time before the onset of deuterium fusion. Using the Virial Theorem and the equation for the gravitational potential energy of a sphere with uniform density, an expression for the rate of energy released for a brown dwarf through gravitational contraction. [15 Points]

$$U = -\frac{3GM^2}{5R}, 2K + U = 0$$

$$U = -\frac{3GM^2}{10R}$$

$$L = \frac{dU}{dt} = \frac{3GM^2}{10R^2}\frac{dr}{dt}$$

5. Jupiter undergoes a similar process in our solar system, shrinking at a rate of around 2cm/yr. Because Brown Dwarfs have similar properties and compositions to Jupiter, assume that Novi C12 B goes through the same rate of contraction, what is the radius of Novi C12 B, knowing that its luminosity is $4.5 \cdot 10^{-5}\ L_{\odot}$? [10 Points]

$$L = \frac{dU}{dt} = 4.5 \cdot 10^{-5} L_{\odot} = \frac{3G(51.224M_J)^2}{10R^2} 2\frac{cm}{year} \rightarrow R = 1.168R_J$$

6. How does the effective temperature of Novi C12 B compare to its equilibrium temperature? What does this mean about the energy generation? (Assume Novi C12 B has similar emissivity and albedo to Jupiter, 0.27 and 0.5 respectively) [20 Points]

Energy Emitted = Energy Reflected

$$\epsilon 4\pi\sigma R^2 T^4 = \alpha I A$$

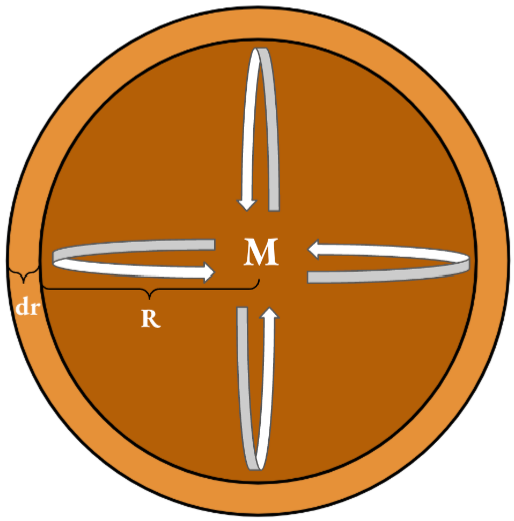
$$L\ \alpha\ M^{3.5} \rightarrow L = (0.579M_{\odot})^{3.5} = 0.148L_{\odot}$$

$$(0.27)4\ \pi\sigma(1.168R_J)^2T^4 = (0.5)(\tfrac{0.148L_{\odot}}{4\pi(1.5AU)^2})(\pi(1.168R_J)^2) \rightarrow T = 164.10K$$

$$4.5 \cdot 10^{-5} L_{\odot} = 4\pi\sigma(1.168R_J)^2T^4 \rightarrow T = 1364.40K$$

3.2 Brown Dwarfs (80 Points)

As mentioned earlier, Brown Dwarfs are very difficult to distinguish from very low-mass stars and even more difficult high-mass planets. The interior of Brown Dwarfs are fully convective. Assume, for simplicity, that there is constant density.



1. From the equation for Hydrostatic Equilibrium, derive an expression for core pressure P_c of a celestial object, in terms of M and R, using the given equations below. [20 Points]

$$\frac{dP}{dr} = -G\frac{m(r)\rho(r)}{r^2}, \quad \int_0^R r\,dr = -\frac{1}{2}R^2$$

$$m(r) = \frac{4}{3}\pi r^3\rho,\ \rho(r) = \rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$\frac{dP}{dr} = -G\frac{(\frac{4}{3}\pi r^3\rho)\rho}{r^2} = \frac{G}{r^2}(\frac{4}{3}\pi r^3(\frac{M}{\frac{4}{3}\pi R^3})^2) = -\frac{3GM^2}{4\pi R^6}r$$

$$\int_R^0 dP = \int_R^0 -\frac{3GM^2}{4\pi R^6}rdr = -\frac{3GM^2}{4\pi R^6}\int_R^0 rdr \rightarrow P_c = \frac{3GM^2}{8\pi R^4}$$

2. What unit is P_c in? Find P_c in terms of Solar Mass $(M/M_\odot)$ and Solar Radii $(R/R_\odot)$. If you could not answer the previous question, use $P_c = \frac{GM^2}{16R^4}$ for half points. [15 Points]

$$P_c = \frac{3GM^2}{8\pi R^4} = \frac{3G(\frac{M}{M_\odot}\cdot 1.989\cdot 10^{30}kg)^2}{8\pi(\frac{R}{R_\odot}\cdot 6.967\cdot 10^8 m)^4} \rightarrow 1.338\cdot 10^{14}(\frac{M}{M_\odot})^2(\frac{R}{R_\odot})^{-4}[Pa]$$

$$P_c = \frac{GM^2}{16R^4} = \frac{3G(\frac{M}{M_\odot}\cdot 1.989\cdot 10^{30}kg)^2}{8\pi(\frac{R}{R_\odot}\cdot 6.967\cdot 10^8 m)^4} \rightarrow 7.00\cdot 10^{13}(\frac{M}{M_\odot})^2(\frac{R}{R_\odot})^{-4}[Pa]$$

3. Using the equation of state below for stars, estimate the core temperature of the lowest mass stars with a mass of 0.08 $M_\odot$ and radius of 0.1 $R_\odot$. Assume the star is fully atomic hydrogen and has uniform density. [15 Points]

$$P\mu m_H = \rho kT, \quad m_H = 1.674 \cdot 10^{-27} kg, \quad \mu = 1$$

$$P_c = 1.338 \cdot 10^{14}(0.08\frac{M}{M_\odot})^2(0.1\frac{R}{R_\odot})^{-4}[Pa] \rightarrow P_c = 8.563 \cdot 10^{15} Pa$$

$$(8.563 \cdot 10^{15} Pa)(1)(1.674 \cdot 10^{-27} kg) = \bar{\rho} k T_c \rightarrow T_c = 9.25 \cdot 10^6 K$$

4. For the equation of state on the star above, it is assumed that the mean molecular weight μ (in terms of m_H) is 1. Based on the star's mass (and therefore, other properties), why are we sure that $\mu = 1$ is a reasonable approximation? [10 Points]

Because the star is just on the limit between a Brown Dwarf and Red Dwarf, the star has a relatively low temperature and is unable to ionize hydrogen (for which 0.5).

5. Why is it a reasonable approximation that the star in Question 2c is fully hydrogen? [10 Points]

(a) Because the star has such a low mass (0.08 Solar Masses), these stars fuse their hydrogen very slowly and have very long lifespans.

(b) The star's low mass enables it to have a very long lifespan (5 trillion years). This means it likely has survived since the beginning of the universe (Population II/III stars), when metals were sparse and mostly hydrogen existed.

6. Where does our core pressure model fail? [10 Points]

It fails at lower mass stars because the average density of lower mass stars are higher than higher mass stars. Because we assumed an average density, this means that our model calculates that lower-mass stars have a higher central pressure than higher-mass stars (obviously not the case). In reality, stars absolutely do not have a uniform density. (The model still works relatively logically for core temperatures between star types though)

3.3 70 Ophiuchi (80 Points)

Seven score and a decade before the first exoplanet was discovered in the 90s, astronomers already postulated methods of detecting exoplanets. In 1855, William Stephen Jacob of Madras Observatory asserted that perturbations in binary star system 70 Ophiuchi represented claims of an unseen exoplanet. It is now known that these observations were erroneous. Using the information below, answer the following.

Combined semi-major axis: 23.328 AU, e = 0.5

1. What method of detection was Jacob attempting to utilize? [5 Points]

   Astrometry

2. Famed German astronomer Friedrich Bessel had previously used the method in 1844 to detect what formerly unseen companion around which famous bright star? [10 Points]

   Sirius B

3. How far apart are 70 Oph A and 70 Oph B? [10 Points]

$$\theta[rads] = \frac{diameter}{distance} \rightarrow \theta[arcsecs] = 206265\frac{diameter}{distance}$$

$$6.7\text{”} = 206265\frac{diameter[m]}{16.7[ly] \cdot 9.46 \cdot 10^{15}[m]}, d = 34.325AU$$

4. Using the masses of 0.90 M and 0.70 M for 70 Oph A and 70 Oph B respectively, determine at what distance from 70 Oph A the gravitational forces from 70 Oph A and B would balance. What is this point called? Is it stable for an exoplanet? [15 Points]

$$\frac{Gm_1}{x^2} = \frac{Gm_2}{(R-x)^2}$$

$$\frac{G(0.90 \cdot 1.989 \cdot 10^{30}[kg])}{x^2} = \frac{G(0.90 \cdot 1.989 \cdot 10^{30}[kg]}{(34.325 \cdot 1.496 \cdot 10^{11}[m] - x)^2}$$

x = 18.239 AU, corresponds to **Lagrange Point 1**, which is **unstable for an orbit.**

5. 70 Ophiuchi is currently near apoapsis for the previous question. What is the distance from 70 Oph A where gravitational forces balance at periapsis? What does this distance mean for an exoplanet? [20 Points]

Closest approach distance: 23.328 [AU](1-0.5) = 11.664 [AU]

$$\frac{G(0.90 \cdot 1.989 \cdot 10^{30}[kg]}{x^2} = \frac{G(0.90 \cdot 1.989 \cdot 10^{30}[kg]}{11.664 \cdot 1.496 \cdot 10^{11}[m] - x)^2}$$

x = 6.198 AU, maximum distance from 70 Ophiuchi A an exoplanet can have a stable orbit.

6. In 1943, astronomers Dirk Reuyl and Erik Holberg claimed to have indirectly observed a planet of theoretical mass between 8.38 MJ , 12.58 MJ with a 17 year period around 70 Oph A. Assuming the planet's mass to be negligible (it probably isn't) and that it is in a circular orbit, determine if the planet is in a stable orbit. Note that the gravitational force from 70 Oph B should not be negligible. Use the periapsis of both stars for this calculation. [20 Points]

$F_{net} = ma_{net}$

Net acceleration on planet is the combined acceleration towards 70 Oph A and 70 Oph B. We are orbiting around 70 Oph B though:

$$a_{net} = \frac{Gm_A}{{r_a}^2} - \frac{Gm_B}{{r_B}^2} = \frac{{v_{planet}}^2}{r_A}, v = \frac{2\pi r}{P}, r = r_A + r_B$$

Using periapsis point, where r=11.664 AU:

$$a_{net} = \frac{Gm_A}{{r_A}^2} - \frac{Gm_B}{(r - r_A)^2} = \frac{(\frac{2\pi r_A}{P})^2}{r_A}$$

Plugging SI unit values in and solving for r_A:
$r_A = 5.131$ AU, **Stable Orbit**