

2026 SONT Astronomy C Question Submission - Key

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Under Pressure — Pressure waves inside variable stars are a critical factor into how their pulsations function. This subfield of astrophysics is called asteroseismology. These waves are driven by phenomena like the κ -mechanism, preventing the pulsation waves from being damped each cycle.

61. Suppose the pulsation period Π_0 of a star depends on its average radius R , pressure P , and density ρ in the form

$$\Pi_0 = C \cdot R^{n_1} \cdot P^{n_2} \cdot \rho^{n_3}$$

where C is a dimensionless constant. Using dimensional analysis, solve for n_1 , n_2 , and n_3 . That is, find the values of n_1 , n_2 , and n_3 such that the units are consistent with a pulsation period.

$$[T]^1 = C [L]^{n_1} \left[\frac{M}{T^2 L} \right]^{n_2} \left[\frac{M}{L^3} \right]^{n_3}$$

$$\text{M: } 0 = n_2 + n_3$$

$$\text{T: } 1 = -2n_2$$

$$\text{L: } 0 = n_1 - n_2 - 3n_3$$

$$\Pi_0 = CR^1 P^{-1/2} \rho^{1/2}$$

$$n_1 = 1, \quad n_2 = -\frac{1}{2}, \quad n_3 = \frac{1}{2}$$

The formula we derived is a good initial model. To obtain a closer estimate, we assume the star has uniform density and derive the expression

$$\Pi_0 \approx \sqrt{\frac{3\pi}{2G\gamma\rho}} \quad \text{where } \gamma = \frac{5}{3}.$$

62. RS Puppis is a Classical Cepheid located ~ 6000 ly away, with a mass of $9.2 M_\odot$ and average radius of $191 R_\odot$. Using our assumed pulsation model, what is the pulsation period, in days, for RS Puppis? What is the range of known Cepheid periods, and is this within it?

$$\rho = \frac{9.2 M_\odot \cdot 1.989 \cdot 10^{30} \text{ kg}/M_\odot}{\frac{4}{3}\pi(191 R_\odot \cdot 6.957 \cdot 10^8 \text{ m}/R_\odot)^3} = 1.862 \cdot 10^{-3} \frac{\text{kg}}{\text{m}^3}$$

$$\Pi_0(\rho) \rightarrow 55.2 \text{ days}$$

Classical Cepheids have periods from a few days to a few weeks (1-100 days). Our value is within the range. RS Puppis has an actual period of ~ 41.5 days,

63. ZZ Ceti stars are pulsating white dwarfs that have cooled to a temperature that allows for partial ionization of hydrogen. How would the pulsation period of ZZ Ceti stars compare to that of Cepheids? Justify your answer using the equation above.

Since $\Pi_0 \propto 1/\sqrt{\rho}$ and white dwarfs have a greater density than Cepheid stars, ZZ Ceti stars would have a shorter period.

A period-average (visible) absolute magnitude relationship for Classical Cepheids is

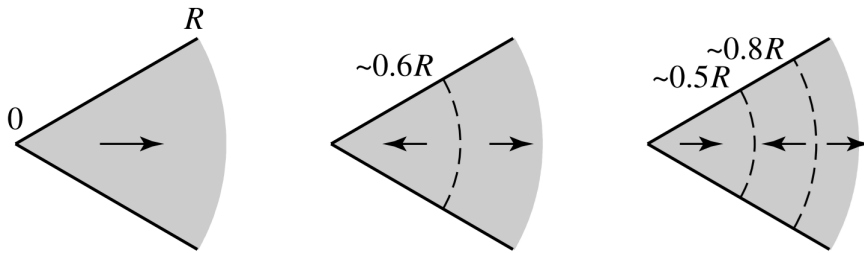
$$\bar{M}_V = -2.43[\log_{10}(P) - 1] - 4.05$$

where P is in days.

64. Using the period in days from Question 62 and other known information, find the average surface temperature of RS Puppis, in Kelvin.

$$\begin{aligned} \bar{M}_V &= -2.43[\log_{10}(55.2 \text{ Days}) - 1] - 4.05 = -5.85 \\ 4.74 - (-5.85) &= 2.5 \log_{10} \left(\frac{L}{L_\odot} \right), \quad L = 17218.69 L_\odot \\ 17218.69 L_\odot &= (191 R_\odot)^2 \left(\frac{T}{T_{eff,\odot}} \right)^4, \quad T_{eff,\odot} = 5778 K \\ T &= 4890 K, [4640, 5140] K \end{aligned}$$

Our formula $\Pi_0(\rho)$ only represents variable stars pulsating in the fundamental mode—which most Cepheids pulsate in. The diagrams below present the fundamental (left), first overtone (center), and second overtone (right) pulsation modes, where multiple regions of the star could be expanding and contracting at once (indicated by arrows). Depending on the mode, there are nodal line(s) (dashed line), a radius where the stellar gas does not expand nor contract.



65. At higher overtones, would you expect the pulsation period of Cepheids to increase or decrease? Why?

Pulsation period would decrease at higher overtones, since the acoustic distance between nodes decreases with each increasing overtone.

66. Provide a reason why the distance from the core of the star to the first nodal line tends to be greater than the distance between the other nodal lines.

The distance from the center of the star to the first nodal line is always the region with the highest sound speed, so acoustic waves travel the farthest over a given pulsation period.